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Abstract

So far the literature on robust monetary policy has used, without exception, very simplified economic models, and with only one exception, assuming closed economy. This paper evaluates topics that are in the research agenda in a more complete and complex set of (open economy) economic models applying the Hansen-Sargent robust control approach. In particular, we are interested in answering the following questions: is the optimal robust monetary policy more aggressive than the standard optimal rational expectation (RE) monetary policy? Are the variances of output and inflation magnified under optimal robust monetary policy? And, more generally, does the introduction of optimal robust monetary policy alter some general conclusions that are established based on the standard RE optimal monetary policy?

In the context of our reference model, we found that the answer to the first two questions is: not necessarily; and to the third one is yes. The answer to the first two questions contradict some previous findings and suggests an obvious conclusion: we cannot generalize conclusions about robust monetary policy using particular models, or in other words, most of the conclusions arrived in this paper as well as in many others (that pretend to be general) are model specific.

Keywords: Hansen-Sargent robust control approach, monetary policy, small open economy, simulation.

JEL classification codes: F41, E52, C61.

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> For all the abundance of competing models in economic research, the agents who populate them are, as a rule, fully devoted to the one model in which they are cast. They know everything about their model (including parameter values) and want to know nothing about any other. All their uncertainty is concentrated on the stochastic elements of the model, which, under the assumption of rational expectations, coincides with the data generating process. Giordani and Söderlind (2004)

1 Introduction

The Hansen-Sargen robust control approach to the economic policy (Hansen and Sargent 2000, 2001 and 2003) is a new instrument that is currently available to overcome one of the main limitations of the current literature in optimal monetary policy (Clarida *et al.* 1999, Svensson 2000), namely, the assumption that the policymaker and the private agents have a perfect knowledge of the economic model that it is in place or at least that they know the objective probability that the model become true.

This problem is not new in the literature, earlier references to non quantifiable uncertainty in models can be found in Knight (1921), Keynes (1936) and Friedman (1959), among others.[1] However the instruments to deal with it were not available until the last decade and, in a more articulated way, until the last two or three years.

So far the literature on robust monetary policy has been based on very small and simplified economic models, and with the only exception of Leitemo and Söderström (2004a) and (2004b), with the assumption of closed economy. The reason lies on the complexity of the techniques that were available. In some grounds this is still the case, but thanks to the contribution of Giordani and Södelind (2004) there are now more easy friendly programs available to work.

This paper takes advantage of the new techniques and programs that are now available to evaluate some questions, that are in the agenda, in a more complete and complex economic model.

[1] See Hansen and Sargent (2000) and (2001) for a historical description of the discussion.

We want to test if the optimal robust monetary policy is more aggressive than the standard optimal rational expectation (RE) monetary policy and if the variances of output and inflation are magnified under robust policy, and also to evaluate how the introduction of optimal robust monetary policy can alter some of the conclusions that we have using standard RE optimal monetary policy. These are important topics that the literature in robust monetary policy is dealing with[2] and are the driving force of this paper.

In the next section, 2, we briefly discuss the history of the robust control theory and review the literature on robust monetary policy. Section 3 is devoted to the conceptual presentation of the robust control technique. In section 4 we discuss in detail the model employed in this paper, whereas in section 5 we present some results. Finally, in section 6 we draw some conclusions.

2 Previous literature on robust control policy

2.1 Brief history

No one could doubt the great advance that the introduction of rational expectations represented to the economic science development in the last three decades, but the concept of rationality was not yet fully exploited in economic modelling (Marcellino and Salmon, 2002). The agents do their best with the available information, but surprisingly, the set of information that they have in most of the cases includes only one economic model! They are perfectly ignorant about any other competing view of the world; such assumption is at least unsatisfactory. But even in the cases where they have model uncertainty, under rational expectations, it is assumed that they know the "correct" probability of the different events, in other words, the personal or subjective probabilities coincides with the objective ones. The last problem is embodied in the narrow concept of rationality that rational expectations represent (Hansen and Sargent, 2000 and 2001).

The concern for model misspecification in the design of monetary policy is not new, both Keynes and Friedman expressed doubts in the effectiveness of monetary policy for this

[2] See for instance, Giordani and Söderlind (2004), Leitemo and Söderström (2004a), (2004b), Giannoni (2002) and Onatski and Stock (2002).

reason. The former was concerned with the inelasticity and fundamentally with the instability in the relationship between investment and the marginal efficiency of the capital. "Animal spirits" was not more than a metaphor for model uncertainty. The latter feared that the policymaker knows too little[3] of the economic structure as to conduce fine tuning monetary policy. But the formalization of these fears not comes until the last decade and in a more systematic way until the last three years.[4]

But, what explains the delay between the identification of the problem and its formalization in a convincing way? Basically, the answer is the unavailability and acknowledge of techniques to deal with the problem.[5] The optimal control theory was already available, but not the robust control theory. It is interesting to reproduce the discussion about the applicability of optimal control techniques to practical policy problems in the UK in the 70s to illustrate this point.

In the early 1976 the UK Parliament approved the creation of a Committee on Policy Optimisation with a commission to report to the Treasure on "the present state of development of optimal control techniques as applied to macroeconomic policy. To make recommendations concerning the feasibility and value of applying these techniques within Her Majesty's Treasury". Shupp (1979), p. 111.

The final statement of the committee describes one of the main limitations of the traditional optimal control theory, the inability to deal with uncertainty:

> "The report describes as a basic source of uncertainty 'the decision-maker's ignorance as to which of all possible economic models best describes the working of the economic system.' ... 'It is well known that economists do not always agree with each other as to how exactly the economy functions.' The

[3] Too little at the point to not allow them to assign probabilities to events, this is the same uncertainty that Keynes refers to.

[4] In February 2002 the journal Macroeconomic Dynamics devoted a special issue to Robust and Risk Sensitive Decision Theory. In this issue we can found Giannoni (2002) and Onatski and Stock (2002) which are among the first papers on robust monetary policy.

[5] The Bayesian techniques, as well as others more or less ad-hoc approaches, were available before the appearance of the robust control techniques in the 90s, but as Onatski and Stock (2002), p.86, pointed out "this approach (the Bayesian) has important limitations, perhaps accounting for the infrequency with which formal decision theoretic methods are used in the actual conduct of monetary policy." However, Sims (2001) had an alternative view (a Bayesian one) and critiques the robust control literature. In section 3 we will illustrate the difference between these two approaches.

> Committee report that the decision-maker 'has to choose among competing models, and does not know which, if any, of the models is the correct one. Control theory has no direct contribution to make to the resolution of this difficulty.'" Johansen (1979), p. 107.

The solution of this problem emerges in the mid 80s with the development of the robust control techniques (Bernhard, 2002), mainly in the engineering literature.[6] The first article that used a related approach in economics seems to be Gilboa and Schmeidler (1989), they use this approach to propose a solution to the famous Elsberg's paradox of expected utility. However, the introduction of the robust control for the main door in economics and especially in macroeconomics began in the late 90s. Lars Hansen and Thomas Sargent are among the precursors, and their objective has been: "to import, adapt, and extend some of the robust control methods to build models of economic agents who experience model ambiguity." (Hansen and Sargent, 2001, p.524).

2.2 Brief review of related literature[7]

As we pointed out before, the introduction of the robust control in the discussion of monetary policy is largely a phenomenon of the last three years.[8] Onastki and Stock (2002) evaluate robust monetary policy in very simple and closed economy macroeconomic model: a backward looking expectations Phillips and demand curve. They found that in most cases (by type of uncertainty) the robust monetary policy is more aggressive than the optimal policy without uncertainty. Giannoni (2002) arrives to a similar conclusion in a simple two equation-forward looking-closed economy model (IS-Phillips curve model). The approach of Giannoni is not exactly the same that we will follow in this paper, but is close.

More recently, Leitemo and Söderström (2004b) used for the first time a small open economy (New-Keynesian) model to re-evaluate the statement that the monetary policy is more aggressive under robust optimality than under RE policy. Their approach differs

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[6] In engineering robust control is still a fertile area or research, see for instance Goulart, Kerrigan and Maciejowski (2005), Björnberg and Diehl (2004) and Löfber (2003).

[7] The literature on robust policy is huge if we include all the approaches (many of them *ad-hoc*) to robustness in economic policy, here we will review only the literature that is near the Hansen-Sargent methodology.

[8] We can found working paper versions at the end of 90s and beginning of 00s.

from the previous in that they are able to distinguish the uncertainty that the policymaker has over the different equations of the model. They conclude "that an increased central bank preference for robustness can make monetary policy respond more aggressively or more cautiously to shocks, depending on the type of shock and the source of misspecification". They pointed out that probably the results in previous literature that indicated a strong policy response under robust policy relies in the assumption of a closed economy of those models.

Levin and Williams (2003) worked over a shortfall of all the preceding papers, the assumption that the true model lies in the neighbourhood of a reference model. They argue, reasonable, the necessity to consider a more general approach that incorporate in the analysis non-nested competing models. Using three small closed economy alternative models they found that the problem became more complex, and only in especial cases a robust outcome is attainable with simple monetary rules. However, in other cases, as under strict inflation targeting, they were unable to found a robust outcome across models.

3 Hansen-Sargent robust policies

Based on Svensson (2000b) we present in this section a very simple framework that illustrate the main features of the robust control approach.

Suppose that the feasible set of models is M, being m a particular model within this set, and $f \in F$ a particular policy rule that belongs to feasible set F. Call V the expected loss function, then $V(f, m)$ is the expected loss of policy f in model m.

If we know the "true" model of the economy, we can proceed as usual[9] and minimize V with respect to f . However, if we do not know the true model m, which is in general the

[9] See, for instance, Söderlind (1999) or Svensson (2000a). This was also the approach in Aboal and Lorenzo (2005).

case[10], we could wrongly choose monetary policy rules that are more prone to generate bigger losses under alternative models.[11]

In this section we will present the two main approaches used in the literature to deal with model uncertainty, the Bayesian approach and the robust control approach (Hansen-Sargent robust policy approach).

The Bayesian approach assigns (a subjective) probability measure, Θ, on the feasible set of models *M*.[12] The expected loss under this configuration is

$$E_M V(f,m) \equiv \int_{m\in M} V(f;m)d\Theta(m).$$

The problem is to minimize the expected loss with respect to *f*, and the solution, f^B, can be written as

$$f^B = \arg\min_{f\in F} E_M V(f,m).$$

The optimal rule will be a function of *M*, *V* and Θ, $f^B(M, V, \Theta)$.

The main difference in the robust control approach, with respect to the previous approach, is that it does not assign any probability, it works with the maximum loss for any given *f*. We can see the approach as a two step (Min-Max) approach, first we choose the worst model for any given policy rule, $m^*(f) = \arg\max_{n\in M} V(f;m)$, and then, from this set we choose the optimal policy rule, f^R, that minimizes the loss

$$f^R = \arg\min_{f\in F}\max_{m\in M} V(f;m) = \arg\min_{f\in F} V(f;m^*(f)).$$

The partisans of this method[13] argued that is an objective method as it does not rely on any *a priori* assumption. In addition, it is also an easy method to use. As we will see in

[10] The true model (the reality) is so complex that we always need simplified assumptions that we hope, in the best case, do not bias our conclusions (with respect to those that could be extracted with the true model). Therefore, by definition our model is not the "true" model.
[11] As Giordani and Söderlind (2004) pointed out the classical application in engineering is "to program a rocket so that it will get very close to the target even if the law of motion is not correctly specified, rather than be on the target if the law of motion is exactly right but go completely astray otherwise."
[12] For an application of this approach see Wieland (2000).

subsection 4.2 to make the analysis robust only requires choosing the value of one parameter.

On the other side, the critics pointed out that when the worst possible model is on the boundary of the feasible set of models, the highly unlikely models can dominate the outcome under robust policy (Svensson, 2000b).[14] Moreover, Sims (2001) is uncomfortable with some aspects of the approach, in particular he pointed out that "it does not always keep the normative analysis of decision making distinct from its descriptive analysis", and in some applications it confuse the tree and the forest, in the sense that unimportant uncertainties (about parameters for example) and important ones tend to be treated as the same.

4 The model

4.1 The Small Open and Financially Vulnerable Economy

The model that is presented in this section is almost identical to that presented in Svensson (2000a)[15], the only difference is that we replace the equation (14) in the Svensson model for the equation (15) in our model. Svensson (2000a) made the ad-hoc assumption that the risk premium follow an AR(1) process. Because we are interested in the features of a partially dollarized economy which is financially fragile (i.e. that are incapable of smoothing out sudden changes in their external financial needs that follows a currency depreciation), we replace this *ad-hoc* assumption by the as well *ad-hoc* assumption that the risk premium depends negatively both with respect to output expressed in terms of foreign goods and to external output.[16] We will not discuss in detail the topic of financially vulnerable economies, but a good reference is Calvo (2000).

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[13] See for instance Giordani and Söderlind (2004).
[14] Giordani and Söderlind (2004) reply this critic.
[15] For some microfundations of the model see the working paper version of the article: Svensson (1998).
[16] We can found justifications for the assumption that the risk premium depends on fundaments, in theoretical models like, Cespedes et al. (2004), Cook (2004) and Choi and Cook (2004) which emphasise the balance sheet effect of the devaluations. This assumption can be as well sustained in empirical grounds, see for instance Eichengreen and Mody (1998) and Berganza et al. (2004) in general, Aronovich (1999) for Latin American countries and Larzabal et al. (2001) for the Uruguayan case.

i. Phillips curve

The forward-looking Phillips curve is:

$$(1) \quad \pi_{t+2} = \alpha_\pi \pi_{t+1} + (1-\alpha_\pi)\pi_{t+3/t} + \alpha_y \left[y_{t+2/t} + \beta_y (y_{t+1} - y_{t+1/t}) \right] + \alpha_q q_{t+2/t} + \varepsilon_{t+2},$$

where for any x, $x_{t+\tau/t}$ is the rational expectation of variable x in period $t+\tau$ with information until t, π_t is the inflation in domestic produced goods in period t, expressed as a log deviation from its mean, y_t is the output gap, defined as

$$(2) \quad y_t = y_t^d - y_t^n,$$

where y^d and y^n are the aggregate demand and the natural level of output expressed in logarithms.

The natural level of output follows a stationary AR(1) process,

$$(3) \quad y_{t+1}^n = \gamma_y^n y_t^n + \eta_{t+1}^n,$$

where $0 < \gamma^n_y < 1$ is a (persistence) parameter and η_t^n is a white noise perturbation (productivity shock).

The real exchange rate, q_t, is defined as

$$(4) \quad q_t = s_t + p_t^* - p_t,$$

where p_t and p_t^* are the logarithm of internal and external prices, s_t is the nominal exchange rate, all variables are expressed as a deviation of their tendencies, ε_{t+2} is a white noise stochastic process that represent inflation shocks or cost push shocks. The coefficients α_π, β_y, α_y, α_q are positive constants, the first two are restricted to be less than one.

Given the definition of the variables, equation (1) shows that the inflation in domestic goods depends on lagged inflation, expected inflation, output gap and expected real exchange rate. The later enters in the equation because it is a measure of the expected costs of the imported inputs in terms of domestic goods, which affects the internal prices.

CPI inflation, π_t^c, can be expressed as a weighted average of domestic and foreign inflation,

(5) $\pi_t^c = (1-\omega)\pi_t + \omega\pi_t^f = \pi_t + \omega(q_t - q_{t-1}),$

where π_t^f is domestic currency foreign inflation (in imported goods), and is defined as

(6) $\pi_t^f = p_t^f - p_{t-1}^f = \pi_t^* + s_t - s_{t-1} = \pi_t + q_t - q_{t-1},$ where $p_t^f = p_t^* + s_t$ y $\pi_t^* = p_t^* - p_{t-1}^*$

again, foreign prices are expresed in log and as a deviation of the appropriate tendency.

ii. Aggregate demand curve

The forward looking demand curve is

(7) $y_{t+1} = \beta_y y_t - \beta_\rho \rho_{t+1/t} + \beta_{y*} y_{t+1/t}^* + \beta_q q_{t+1/t} - (\gamma_y^n - \beta_y) y_t^n + \eta_{t+1}^d - \eta_{t+1}^n,$

where y_t^* is the external output gap, ρ_t is the sum of current and expected future (deviations from a constant mean) real interest rates.[17] All the coefficients are non negatives and $\beta_y < 1$; η_t^d is a zero mean demand shock.

The real interest rate is defined as

(8) $r_t = i_t - \pi_{t+1/t},$

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[17] $\rho_t = \sum_{\tau=0}^{\infty} r_{t+\tau/t}$, $r_t^T = \frac{1}{T}\sum_{\tau=0}^{\infty} r_{t+\tau/t}$, where r_t^T is the rate of a long real zero-coupon bond, see Svensson (1998) for details.

where i_t is the nominal interest rate, the policy instrument of the central bank.

Therefore, the aggregate demand depends on (the lagged) output gap, expected real interest rates, expected foreign output gap, natural output and productivity and demand shocks.

The exchange rate fulfils the parity condition

(9) $i_t - i_t^* = s_{t+1/t} - s_t + \varphi_t,$

where i_t^* is the nominal external interest rate and φ_t is a risk premium. Taken in account (4), (9) can be expressed as

(10) $q_{t+1/t} = q_t + i_t - \pi_{t+1/t} - i_t^* + \pi_{t+1/t}^* - \varphi_t.$

iii. Foreign variables

It is assumed that the foreign inflation and output gap follows stationary AR(1) stochastic process, and the nominal interest a Taylor rule,

(11) $y_{t+1}^* = \gamma_{y*} y_t^* + \varepsilon_{t+1}^*$

(12) $\pi_{t+1}^* = \gamma_{\pi*} \pi_t^* + \eta_{t+1}^*$

(13) $i_t^* = f_{\pi*} \pi_t^* + f_{y*} y_t^* + \xi_{it}^*,$

where $\gamma_{y*}, \gamma_{\pi*}, f_{\pi*}$ y f_{y*} are constant positive parameters and $\varepsilon_{t+1}^*, \eta_{t+1}^*$ y ξ_{it}^* zero mean perturbations.

iv. Risk premium

It is assumed that the (first difference) risk premium depends on two measures of the capacity of the country to pay the debt service (the debt was emitted in “foreign goods price”), the net exports, nx_t, and the output in terms of foreign goods,

$$(14) \quad \varphi_{t+1} - \varphi_t = -\psi_1 nx_t - \psi_2 (y_t - q_t) + \xi_{\varphi t+1}$$

where ψ_j (j = 1,2) are parameters that we expect to be positive in a financially vulnerable economy and $\xi_{\varphi t+1}$ is zero mean risk premium shock.

Assuming that net exports depends on foreign output we can rewrite (14) as

$$(15) \quad \varphi_{t+1} = \varphi_t - \psi_{y*} y_t^* - \psi_{y-q} (y_t - q_t) + \xi_{\varphi t+1} .$$

v. Loss function

The loss function, L_t, is postulated in terms of output gap, CPI inflation and smoothing terms for the exchange rate and the interest rate,

$$(16) \quad L_t = \mu_c \pi_t^{c2} + \lambda y_t^2 + d(s_t - s_{t-1})^2 + \upsilon_i (i_t - i_{t-1})^2 .$$

where μ_c, λ, d and υ_i are preference parameters.

The third component in the loss function intents to reflect the concern of the monetary authority for abrupt changes in perhaps the most important price in a partially dollarized-financially vulnerable economy.

4.2 The model in state-space format

The problem that we have to deal with can be represented in a state-space format as follows,

(17) $$\min_{\{i\}_0^\infty} \max_{\{v\}_1^\infty} J_0 = E_0 \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + u_t' R u_t + 2x_t' U u_t - \theta v_{t+1}' v_{t+1}),$$

s.t.

(18) $$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + Bu_t + C\left[v_{t+1} + \varepsilon_{t+1}\right], \text{ where } x_t = \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}.$$

The matrices Q, R and U define the weight of the variables in the loss function, β is a discount factor, x_t is state vector composed by n_1 predetermined variables, x_{1t}, and n_2 forward looking variables, x_{2t}, u_t is the vector of instruments of the monetary authority and v_{t+1} are the control variables that allow to modify the model, the parameter θ is the key parameter that measures the degree of robustness required,[18] small values represent a more cautious policymaker (greater fear of misspecification). The system (18) represents the economy, where the matrices A, B and C are matrices of parameters and ε_{t+1} is the vector of shocks of the model (the last n_2 terms in the vector are zero).

In the solution, u_t and v_{t+1} will be linear functions of the state vector x_t

(19) $u_t = -F_u x_t,$

(20) $v_{t+1} = -F_v x_t,$

where F_u and F_v are matrices that will be determined in the optimization process.

In our case the vectors of variables are defined in the following way

$$x_{1t} = (\pi_t, y_t, \pi_t^*, y_t^*, i_t^*, \varphi_t, y_t^n, q_{t-1}, i_{t-1}, \pi_{t+1/t})'$$
$$x_{2t} = (q_t, \rho_t, \pi_{t+2/t})',$$
$$u_t = i_t.$$

The other variables not considered in these vectors are simple identities, with the exception of $i_{t+1/t}$. In annex C of Aboal and Lorenzo (2005) and in annex E of Svensson (1998) is explained how to use the equations to express the system as described above (with minor differences).

[18] See Giordani and Söderlind (2004) for an excellent discussion of the role and the meaning of this parameter in this kind of problems.

In annex A we show the parameters used in the calibration of the model. Most of the parameters come from the work of Moron and Wilkenried (2003). These parameters were estimated using Uruguayan data, and then, reflect the structure of a small open and partial dollarized economy. Do to limited space the main futures of this economy are not discussed, please refer to Moron and Wilkenried (2003), Lorenzo, Aboal and Osimani (2005) and Aboal, Lorenzo and Noya (2002) for some description and additional references.

In annex B and C of Aboal and Lorenzo (2005) is shown some manipulations with the equations to express the model in a state-space format similar that that presented in (17)-(18).[19] This system will be introduced in the optimization programs.[20] In annex C of Aboal and Lorenzo (2005), it is also shown the algorithm used in the optimization programs, which comes from Söderlind (1999).

5 Solving the model for robust and non-robust policies under discretion

In this section we perform to types of exercises both for robust and non-robust monetary policy. First, we compute the variances of output and inflation under different configurations of weights of the different variables in the loss function. Second, we carry out an impulse response analysis.

5.1 Variances

We are interested in the comparison of the variances of the output gap, inflation, real exchange rate and risk premium under different relative weight of the objective variables in the loss function and under robust and non-robust optimal policy rules. These four variables represents perhaps the most important variables in a small open partial dollarized economy, and therefore, we are interested to determine their sensibility (approximated by

[19] See also Svensson (1998).
[20] The Matlab codes are available upon request to author.

their variances) to changes in the preferences of the policymaker both under robust and non-robust policies.

To this end we perform a Monte Carlo simulation experiment with 1000 vectors of independent shocks (randomly generated from a normal distribution with zero mean and variances equal to those presented in annex A). The experiment consist in the calculation of the optimal rule (both robust and non-robust) using different configurations of weights in the loss function, and for each of this rules we calculate the variance of the output and inflation applying over the system the (1000) vectors of shocks (the variances are computed when the system converges to a new equilibrium, and not after a determined number of periods).

As reported by other authors (Giordani and Söderlind, 2004, Leitemo and Söderström, 2004a, 2004b) we distinguish two cases under robust policy: the *worst case model* and the *approximating model*. In the *worst case model* the planner is fully pessimist and chooses the worst model. In contrast, in the *approximating model*, although the policy rule is the same as in the *worse case model* there is no misspecification in the model that we use for the simulations.

More formally, in an equilibrium condition the variables x_{1t} follow a VAR(1) process represented by:

$$x_{1t+1} = Mx_{1t} + C\varepsilon_{t+1},$$

where M is a matrix of parameters and C was already defined. The forward looking variables and the instruments are linear functions of the predetermined variables,

$$\begin{bmatrix} x_{2t} \\ u_t \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} N \\ -F_u \\ -F_v \end{bmatrix} x_{1t},$$

where N is a matrix of parameters.

The difference between the *worst case model* and *approximating model* is only consequence of a different matrix *M*. In the *approximating model*, the reference model is assumed to be the true one in the conformation of the matrix *M*. Therefore, the difference in *M* matrix in one or another case indicates the direction of the fear of misspecification of the planner (see Giordani and Söderlind, 2004).

The results of the simulation are presented in tables 1 and 2.

Some conclusions can be extracted from this exercise. In first place, we have not found that robust policies always generates greater volatility than the RE policies.[21] Moreover, if we compare rational expectations with the worst case model for all four variables and for all the cases analyzed (see tables 1 and 2) only in two cases out of 16 the robust policy increases variances. Furthermore, by comparing RE and the approximating case we reach similar conclusion.

Table 1. Output and inflation variances

Cases	Rational Expectations		Robust			
			Worst case		Approximating	
	σ_π^2	σ_y^2	σ_π^2	σ_y^2	σ_π^2	σ_y^2
Baseline						
$\mu_c=1, \lambda=1, d=0, \nu_i=0$	0.0853	0.4668	0.0154	0.1483	0.0462	0.0269
General						
$\mu_c=1, \lambda=1, d=1, \nu_i=1$	0.0283	0.0237	0.0022	0.0195	0.0006	0.0095
Strict Inflation Targeting						
$\mu_c=1, \lambda=0, d=0, \nu_i=0$	0.0012	0.2945	0.0008	0.2623	0.0222	0.2825
Output Target						
$\mu_c=0, \lambda=1, d=0, \nu_i=0$	0.0879	0.0009	0.0861	0.0009	0.0861	0.0009

Table 2. Real exchange rate and risk premium variances

Cases	Rational Expectations		Robust			
			Worst case		Approximating	
	σ_φ^2	σ_q^2	σ_φ^2	σ_q^2	σ_φ^2	σ_q^2
Baseline						
$\mu_c=1, \lambda=1, d=0, \nu_i=0$	0.0809	0.5293	0.0074	0.4246	0.0133	0.0325
General						
$\mu_c=1, \lambda=1, d=1, \nu_i=1$	0.028	0.0434	0.0415	0.0636	0.0364	0.0222
Strict Inflation Targeting						
$\mu_c=1, \lambda=0, d=0, \nu_i=0$	0.0006	0.3074	0.0004	0.2863	0.0031	0.2921

[21] Giordani and Söderlind (2004) in a context of New Keynesian model concludes: "robustness … increases inflation and output volatility".

Output Target						
μ_c=0, λ=1, d=0, ν_i=0	0.0319	0.008	0.0319	0.008	0.0319	0.008

Second, the variances under the *worst case model* are not always the worst and in most of the cases are not very dissimilar than in the *approximating model*, suggesting that the *worst case model* is not very different from the reference one in our particular case.

Third, the presence of smoothing terms in the loss function (general case) leads to a more stable economy (with respect to baseline case), in all but two cases the variances considered in tables 1 and 2 are reduced drastically. This conclusion is maintained both under robust and non-robust policy.

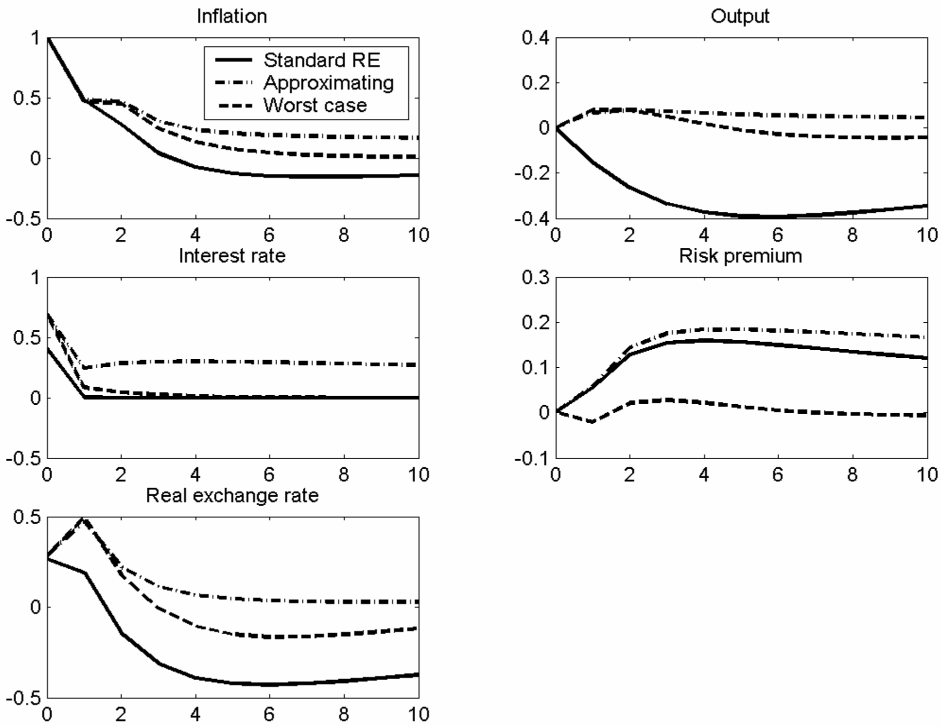
Finally, as expected, in the cases of strict inflation targeting and output targeting, the variance of the target variable is drastically reduced with respect to all other cases and the introduction of robust policy does not alter in general this conclusion. However, we have observed one exception: the general case in the *approximating model* conduce to less inflation volatility than the strict inflation targeting case.

5.2 Impulse response functions

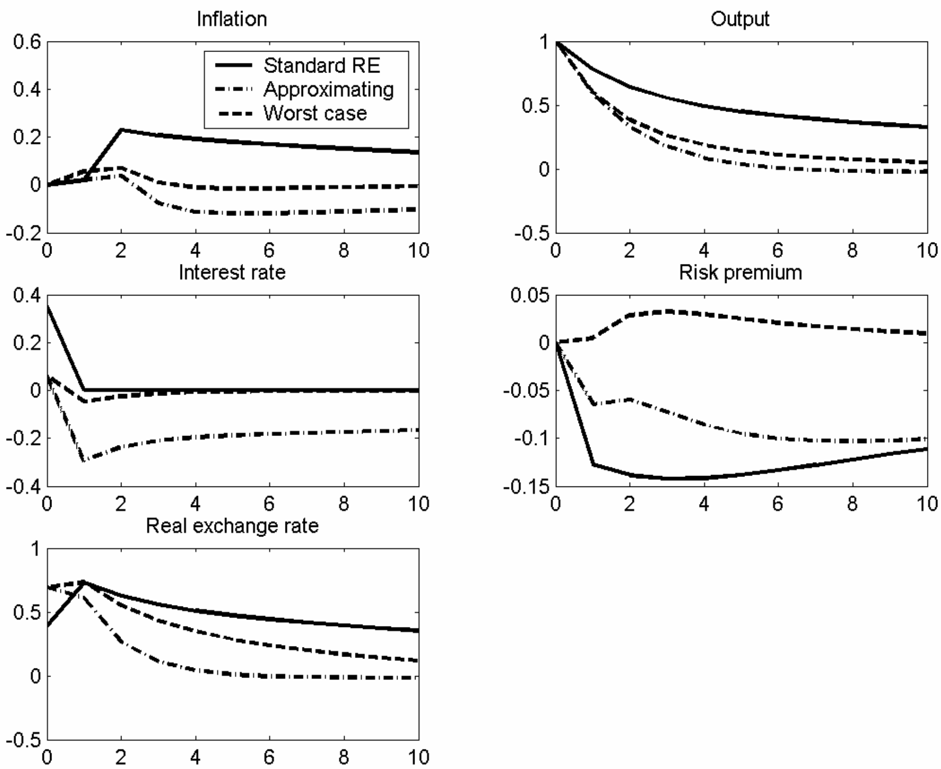
In this subsection we re-evaluate the statement that under robust monetary policy the policymaker generates a stronger monetary policy than under RE policy, in the sense that he reacts strongly to given shocks. This is of particular interest to us because we presume that no general conclusion can be extracted, and that the previous results in the literature rely in the particular structure of the economy. In this paper we have the most complete (and complex) model used up to the date for this kind of exercises. Therefore, we are interested in analyse in comparative terms the results that we can extract from it.

The following two panels show the impulse response exercise performed with (unitary) shocks over inflation and output gap (in the baseline case). As expected, in both cases the optimal reaction implies an increase in the interest rate, trading in this way between the objective of price and output stability.

Panel 1. Impulse response functions following an inflationary shock



Panel 2. Impulse response functions following an output gap shock



In the case of the inflationary shock, following the increase in the interest rate, the real exchange via (10) also increases in the short run and this in turn augment the risk premium that the country faces. The net effect over the output is positive in the case of the robust policy, but negative in the case of RE policy.

Following the output shock, the reaction of the policy in the first period is similar to the previous case and also the effects over the real exchange rate and the risk premium. However, the effect over the latter is reversed quickly under RE policy, and this is the factor that is attenuating the impact of the increased interest rate over the output.

We mentioned before that many authors pointed out the empirical fact that in most cases the robust monetary policy reacts stronger than the standard policy to a given shock. As we can see in panel 1 the conclusion that the robust policy (interest rate) reacts stronger in the first period to a given shock seems to be confirmed, but, if we look at the reaction of the instrument to an output shock (panel 2) the conclusion is exactly the opposite. We perform the same exercise with shocks over foreign inflation, foreign output, international interest rate, risk premium and natural output, in only three cases the robust policy was more aggressive than in the standard rational expectation case (see annex B). Our results are in line with Leitemo and Söderström (2004b) in the sense that the responsiveness of the robust policy depends on the type of shocks.

Leitemo and Söderström (2004b) suggest that this result is associated to the assumption of a closed economy. But we interpret the phenomenon as dependent on the structure of the economy (the model) more than specifically with the single assumption of closed versus open economy. Moreover, we consider that the discussion in these terms is not very fruitful, unless we can perform an analysis with nested models, where the different cases are particular cases of a more general model, which is the basic requisite of a controlled experiment. Otherwise, it is impossible to identify the factors that are behind the aggressiveness of the policy. We can not compare two models that differ in more than one dimension and then assign the different outcomes to only one characteristic.

6 Conclusions

Three questions guided this research in the context of a (reference) model for a small open financially vulnerable economy: is the optimal robust monetary policy more aggressive than the standard optimal rational expectation (RE) monetary policy? Are the variances of output and inflation magnified under optimal robust monetary policy? Can the introduction of optimal robust monetary policy alter some general conclusions that are established under standard RE optimal monetary policy?

The answers to these three questions are the main conclusions of the paper. In first place, our analysis show that the robust monetary policy does not always reacts stronger than the standard RE one.

Second, the robust policy leads to a reduction in the variances of inflation and output as well as in the variance of the real exchange rate and the risk premium in most of the cases considered. These results contradict previous literature; see for instance Giordani and Söderlind (2004).

Finally, at first glance, the variance exercise (tables 1 and 2) suggests that robust control is not very relevant in the sense that the order of cases that leads to a more stable economy (less variances) is not altered by the introduction of robust control. However, based on the impulse response exercise performed in this paper, we observed that the evolution of the variables along time, under different shocks and robust versus non robust policy, are in many cases very different. This leads to the conclusion that the introduction of robust control matters to analyse the impact of shocks over the economy in the short run.

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Anexo A
Parameters used in the calibration of the model

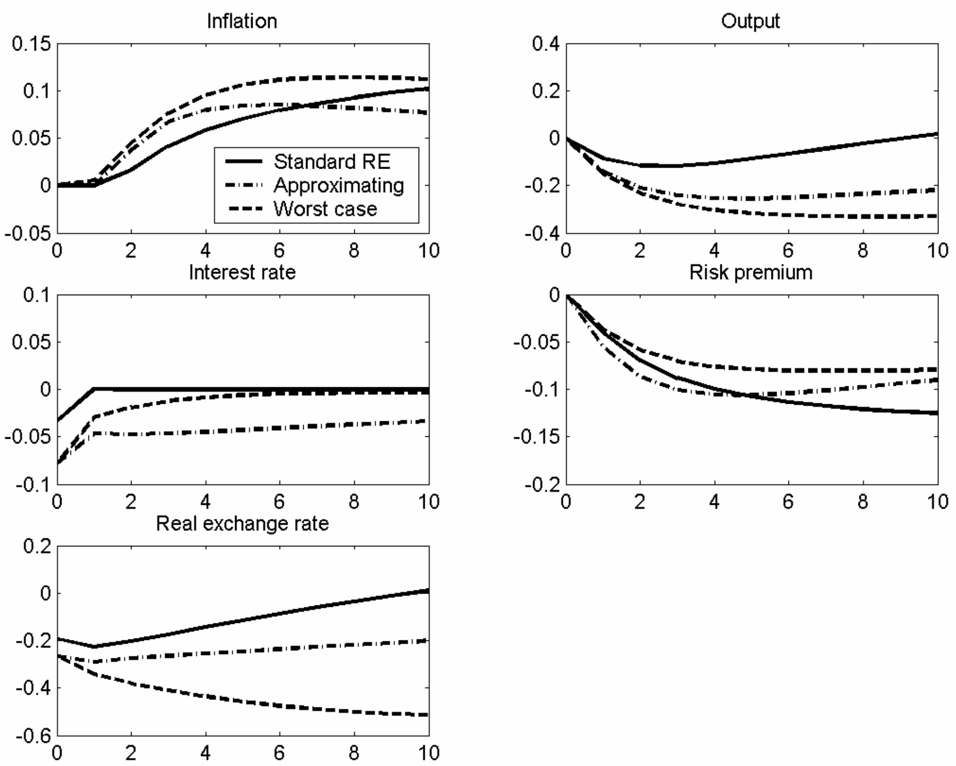
The parameter ω, is from Lorenzo, Aboal and Osimani (2003), β that was set arbitrary and θ adopts the value suggested by Giordani and Södelind (2004). The other parameters aie from Morón and Wilkenried (2003).

Equations	**Parameter values**
Natural output gap	
γ_y^n	0.960
$\sigma(\eta_{t+1}^n)$	0.020
Foreign inflation	
γ_π^*	0.910
$\sigma(\varepsilon_{t+1}^*)$	0.005
Foreign output gap	
γ_y^*	0.790
$\sigma(\eta_{t+1}^*)$	0.005
Foreign Taylor rule	
f_π^*	0.760
f_y^*	0.430
$\sigma(\xi_{it+1}^*)$	0.005
Phillips curve	
α_π	0.480
α_y	0.036
α_q	0.075
$\sigma(\varepsilon_{t+2})$	0.007
Aggregate demand	
β_y	0.556
β_{y*}	0.404
β_q	0.038
β_ρ	0.048
$\sigma(\eta_{t+1}^d)$	0.022
Risk premium	
φ_{y*}	0.542
φ_{y-q}	0.210
$\sigma(\xi_{\varphi,t+1})$	0.012
Others	
ω	0.500
β	0.950
θ	55

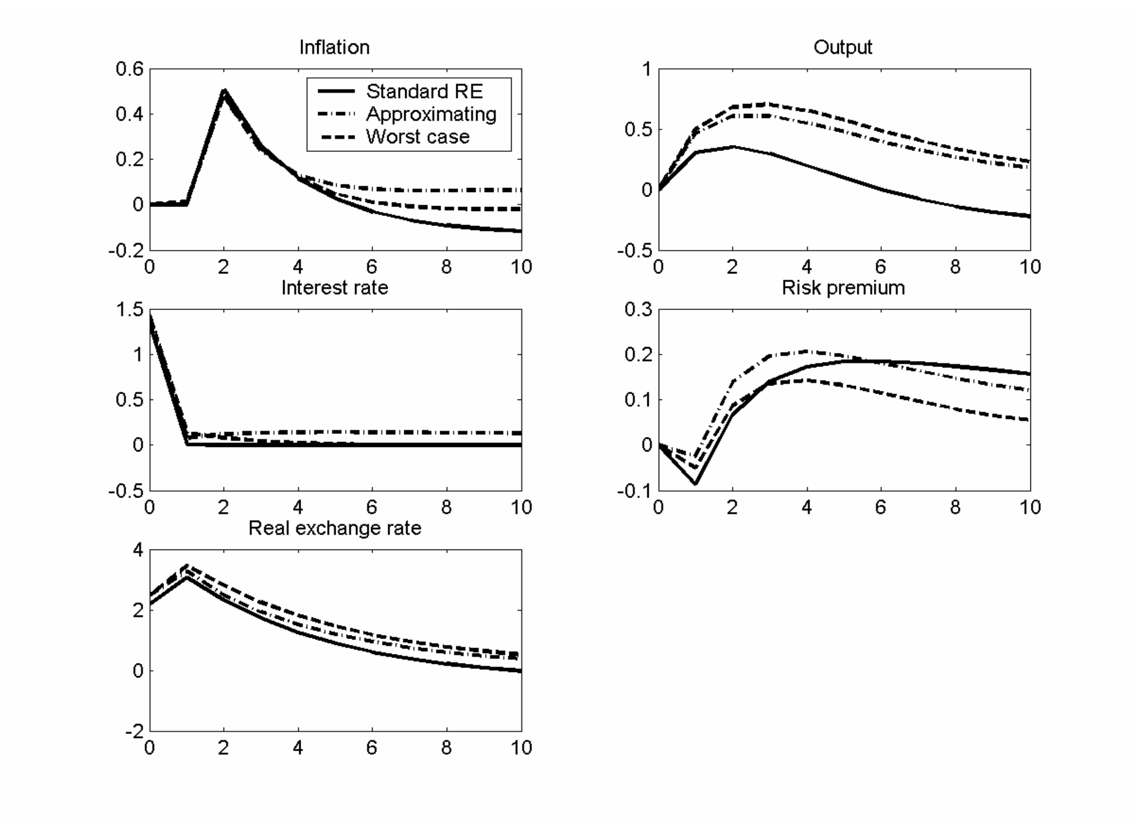
Annex B

Impulse response functions

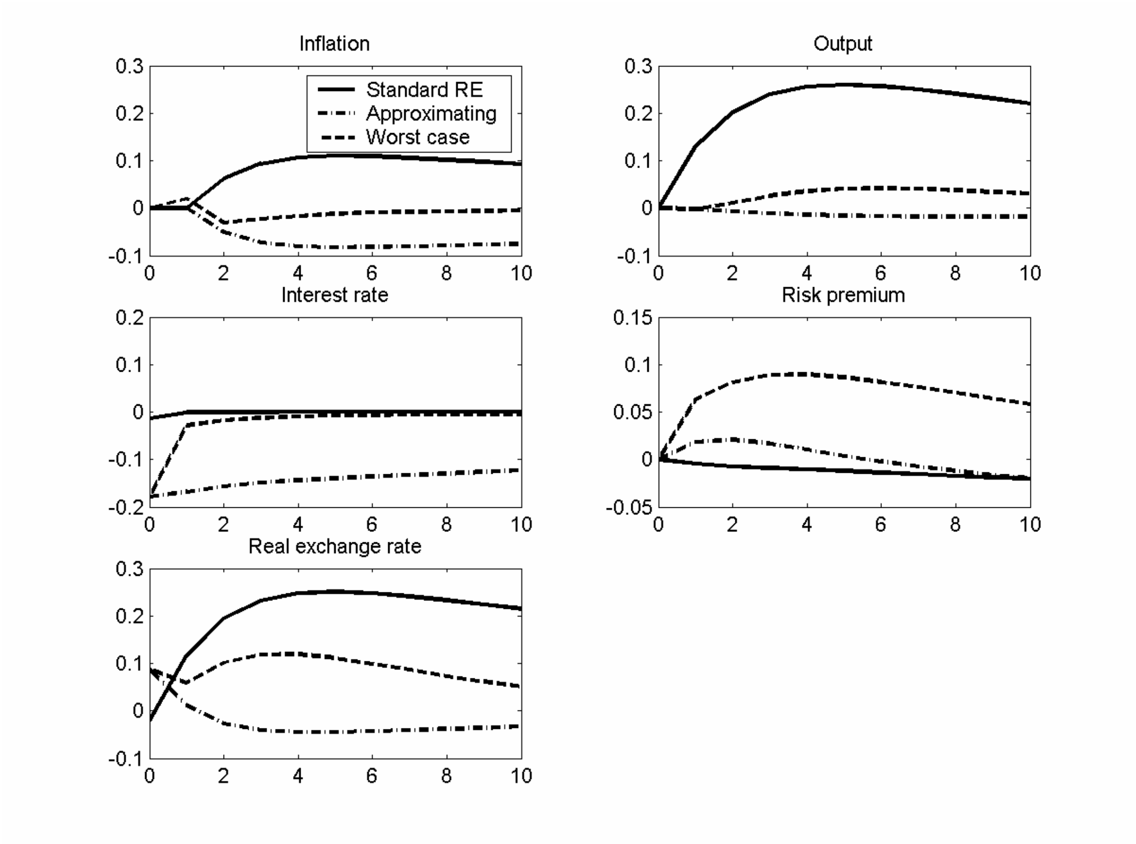
Graph B1. Technological shock (natural output)



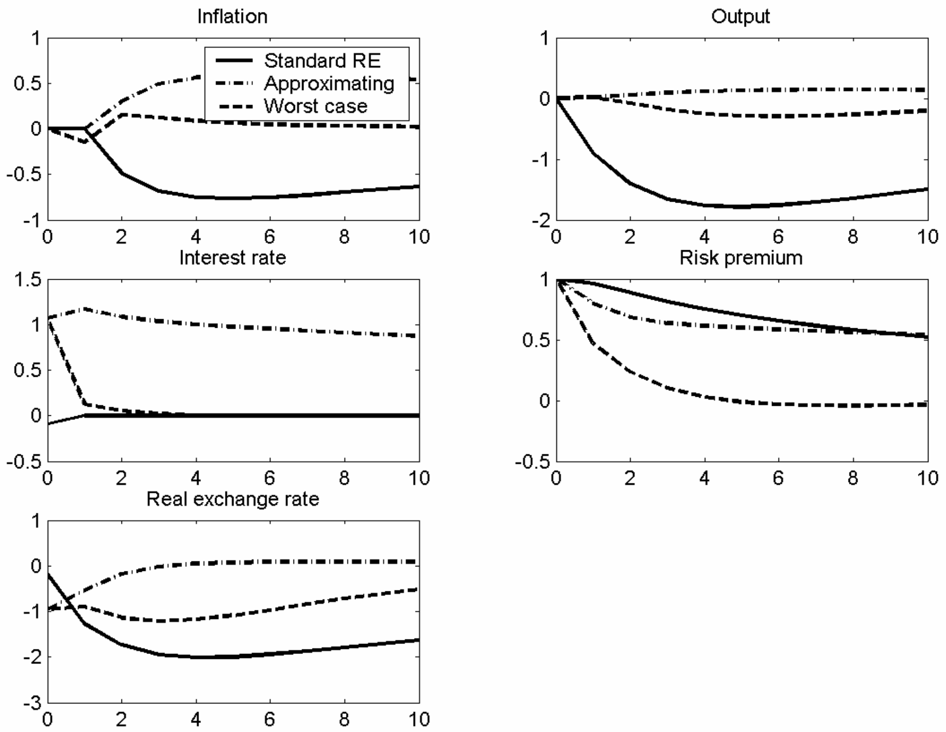
Graph B2. Foreign output shock



Graph B3. Foreign inflation shock



Graph B4. Risk premium shock



Graph B5. International interest rate shock